Quadratic Equations

Key Points:

- A ***quadratic equation*** is an equation containing a second-degree polynomial; for example, $ax^2 + bx + c = 0$, where a, b, and c are real numbers. If $a \neq 0$, it is in standard form.

- Written in standard form, $ax^2 + bx + c = 0$, any quadratic equation can be solved using the ***quadratic formula***, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, and c are real numbers and $a \neq 0$.

- Many quadratic equations can be solved by factoring when the equation has a leading coefficient of 1 or if the equation is a difference of squares. The zero-product property is then used to find solutions.

- Many quadratic equations with a leading coefficient other than 1 can be solved by factoring using the grouping method.

- Another method for solving quadratics is the square root property. The variable is squared. We isolate the squared term and take the square root of both sides of the equation. The solution will yield a positive and negative solution.

- Completing the square is a method of solving quadratic equations when the equation cannot be factored.

- A highly dependable method for solving quadratic equations is the quadratic formula, based on the coefficients and the constant term in the equation.

- The discriminant is used to indicate the nature of the roots that the quadratic equation will yield: real or complex, rational or irrational, and how many of each.

Value of Discriminant	Results
$b^2 - 4ac = 0$	One rational solution (double solution)
$b^2 - 4ac > 0$, perfect square	Two rational solutions
$b^2 - 4ac > 0$, not a perfect square	Two irrational solutions
$b^2 - 4ac < 0$	Two complex solutions

Example: Solve the quadratic equation $x^2 + 5x + 1 = 0$.

Solution:

(1) Identify the coefficients: $a = 1, b = 5, c = 1$.
(2) Use the quadratic formula:

$$\boldsymbol{x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(1)}}{2(1)}}$$
$$\boldsymbol{= \frac{-5 \pm \sqrt{25 - 4}}{2}}$$
$$\boldsymbol{= \frac{-5 \pm \sqrt{21}}{2}}$$

Quadratic Equations Videos

- Solving Quadratic Equations by Factoring
- Solving Quadratic Equations by using the Square Root Property
- Solving the Quadratic Equations by Completing the Square
- Solving Quadratic Equation by using the Quadratic Formula

Practice Exercises

Follow the directions for each exercise below:

1. Solve the quadratic equation by factoring: $\boldsymbol{2x^2 - 7x - 4 = 0}$

2. Solve the quadratic equation by factoring: $\boldsymbol{3x^2 + 18x + 15 = 0}$

3. Solve the quadratic equation by factoring: $\boldsymbol{25x^2 - 9 = 0}$

4. Solve the quadratic equation by factoring: $\boldsymbol{7x^2 - 9x = 0}$

5. Solve the quadratic equation by using the square-root property: $\boldsymbol{x^2 = 49}$

6. Solve the quadratic equation by using the square-root property:

$$\boldsymbol{(x - 4)^2 = 36}$$

7. Solve the quadratic equation by completing the square: $\boldsymbol{x^2 + 8x - 5 = 0}$

8. Solve the quadratic equation by completing the square: $\boldsymbol{4x^2 + 2x - 1 = 0}$

9. Solve the quadratic equation by using the quadratic formula.

If the solutions are not real, state "No real solution:" $\boldsymbol{2x^2 - 5x + 1 = 0}$

10. Solve the quadratic equation by using the quadratic formula.

If the solutions are not real, state "No real solution:" $\boldsymbol{15x^2 - x - 2 = 0}$

11. Solve the quadratic equation by the method of your choice: $\boldsymbol{(x - 2)^2 = 16}$

12. Solve the quadratic equation by the method of your choice: $\boldsymbol{x^2 = 10x + 3}$

Answers:

1. $x = 4, -\frac{1}{2}$

2. $x = -1, -5$

3. $x = \pm\frac{3}{5}$

4. $x = 0, \frac{9}{7}$

5. $x = \pm 7$

6. $x = 10, -2$

7. $x = -8 \pm \sqrt{21}$

8. $x = \frac{-1 \pm \sqrt{5}}{4}$

9. $x = \frac{5 \pm \sqrt{17}}{4}$

10. $x = \frac{2}{5}, -\frac{1}{3}$

11. $x = -2, 6$

12. $x = 5 \pm 2\sqrt{7}$